MACHINE LEARNING FOR REDUCED ORDER MODELING OF NONLINEAR INDUCTORS

Matteo Zorzetto[1], **Riccardo Torchio**[1,2], **Francesco Lucchini**[1], **Paolo di Barba**[3], **Maria Evelina Mognaschi**[3], **Michele Forzan**[1], **Fabrizio Dughiero**[1].

[1] Department of Industrial Engineering, Università degli Studi di Padova, Via Gradenigo 6/A, 35131, Padova, Italy
[2] Department of Information Engineering, Università degli Studi di Padova, Via Gradenigo 6/B, 35131, Padova, Italy
[3]Department of Electrical, Computer, and Biomedical Engineering, University of Pavia, Pavia 27100, Italy
matteo.zorzetto@phd.unipd.it

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Abstract

This paper explores the integration of the Proper Orthogonal Decomposition (POD) algorithm with machine learning to develop a non-intrusive reduced-order model for a nonlinear inductor. Using the Finite Element Method (FEM), an axisymmetric model of a nonlinear inductor is constructed and a dataset consisting of the magnetic vector potential's spatial distribution for different current values is collected. POD is applied to reduce the dataset's dimensionality, while machine learning establishes a mapping between the inputs and the reduced state vector. The resulting surrogate model significantly increases computational efficiency with minimal accuracy loss.

1 Introduction

The Finite Element Method (FEM) is a robust approach for modeling nonlinear magnetic problems by discretizing governing equations into a system of partial differential equations. However, applications like optimization and model predictive control demand reduced computational time and memory usage. Model Order Reduction (MOR) techniques, such as Proper Orthogonal Decomposition (POD) [1], tackle these challenges by projecting the system matrices of the Full Order Model (FOM) onto a lower-dimensional subspace, resulting in a Reduced Order Model (ROM).

MOR techniques are effective for linear problems but face additional complexities when applied to nonlinear systems. Methods like the Discrete Empirical Interpolation Method [2] can yield strong results for nonlinear systems; however, their performance is case-dependent. Moreover, their reliance on intrusive modifications can be challenging, particularly when the FOM is developed using commercial software. An alternative approach leverages machine learning to directly map inputs to the solution, offering greater flexibility but encountering challenges such as the high computational cost of generating training data and managing the high dimensionality of the solution space. Previous research has addressed this by employing the Proper Orthogonal Decomposition (POD) algorithm to reduce solution dimensionality, followed by training machine learning models —such as Gaussian Process Regression (GPR) [3] and Artificial Neural Networks (ANNs) [4]— to map inputs to the reduced solution vectors. In this work, the approach is applied to create a ROM of an inductor wound around a nonlinear magnetic core.

2 Case study

The device under study consists of a multi-turn coil of 100 turns wound around a ferrite half-core typical of proximity sensors, whose dimensions are specified in [5]. A ferrite disc with a diameter of 70 mm and a width of 14.5 mm is positioned at a distance of 1 mm from the half-core. The 3D problem is approached through the magnetostatic problem in axisymmetric coordinates:

$$\nabla \cdot \nu'(\Phi)\nabla\Phi = -\mathrm{J}_{\mathrm{s},\theta}, \quad (1)$$

written in the (r, z, θ) coordinates system. In Equation (1) $\nu' = \frac{1}{r\,\mu_r(\boldsymbol{B})\mu_0}$ is the magnetic reluctivity, dependent on the magnetic flux density $\boldsymbol{B} = \nabla \times \boldsymbol{A}$. $\Phi = rA_\theta$ represents the magnetic vector potential in the θ direction, scaled by the distance r from the axis of symmetry, while $J_{s,\theta}$ represents the source current, assumed uniform in the multiturn coil fed by DC current. Equation (1) is also subject to the Dirichlet boundary condition. Due to the effect of magnetic saturation, as presented in Fig. 1, the concatenated flux grows nonlinearly with the coil current.

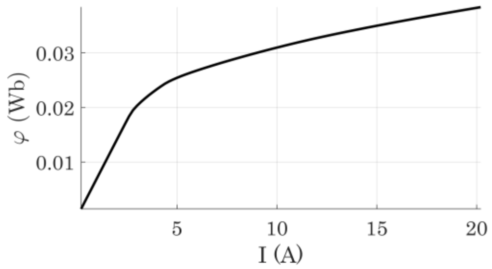

Figure 1: Concatenated flux for different current values.

Equation (1) is discretized using the FEM, obtaining the nonlinear system of equations:

$$\mathbf{K}(\boldsymbol{x})\boldsymbol{x} = \mathbf{B}\boldsymbol{u}. \quad (2)$$

3 Reduced order modeling

The FOM is first simulated for 101 current values, obtaining the nodal distribution of A_θ by shifting $\boldsymbol{x}$. The data collected is then split into a training and validation dataset, obtaining two snapshot matrices of 29 and 72 items respectively. POD is used to find a reduced basis $\boldsymbol{V}$ for the solution by collecting a limited number of singular vectors of the training snapshot matrix, presented in Fig. 2.

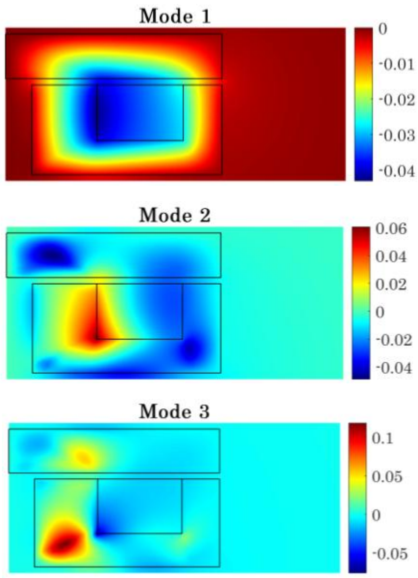

Figure 2: First three modes of the snapshot matrix.

The reduced basis is employed to project the state vector as $\boldsymbol{x} \approx \boldsymbol{V}\boldsymbol{z}$, reducing the number of variables from 2932 to 3. GPR is then employed to find a relationship between the input current and the coefficients $\boldsymbol{z}$. The GPR surrogate is then used to obtain a prediction $\hat{\boldsymbol{z}}$ for new current values, which can be expanded using the basis $\mathbf{V}$ obtaining the nodal distribution of A_θ:

$$\hat{\boldsymbol{x}}(u) = \mathbf{V}\hat{\boldsymbol{z}}(u). \quad (3)$$

As shown in Fig. 3 the surrogate model predicts the POD coefficients, providing an accurate estimate for A_θ, presented for a validation point in Fig.4. The accuracy of the surrogate depends on two factors. The reconstruction error, introduced by the POD can be reduced by increasing the number of modes considered. Moreover, the prediction error associated with the machine learning approach can be tackled by tuning the algorithm and increasing the number of data points.

2 Conclusions

While preliminary results show the effectiveness of the approach, the full paper will adapt the method for a more complex representation of the device. Moreover, different machine learning approaches will be considered.

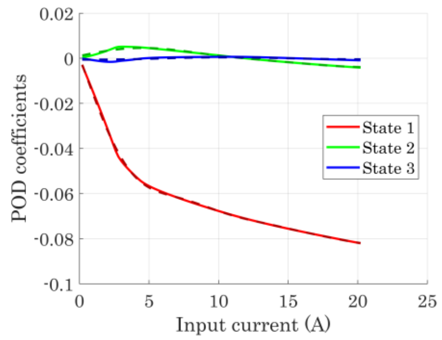

Figure 3: Reduced state vector as a function of the input current, dashed lines represent the model prediction while continuous lines are the true solution.

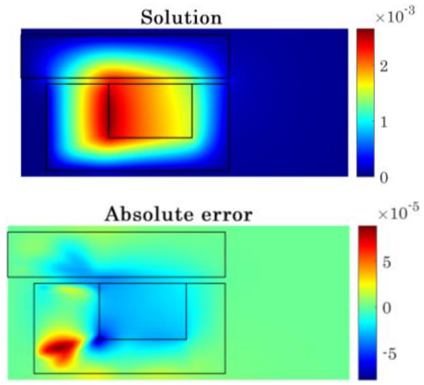

Figure 4: Solution and prediction error for $I = 5.4$ A

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